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## THEORETICAL CONTRIBUTIONS TO THE DEVELOPMENT OF NEW VALUATION MODELS OF FACE MILLING FORCES

BY

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**Abstract.** With this paper one intend to improve the valuation models of face milling forces so that all significant influences to be present at their true value. In support of this we'll consider mainly the cutting force components acting on a tooth and the number of teeth that simultaneously cut. The working conditions, namely the cutting regime, insert's geometry, the nature of material being cut and the cutting environment are present in  $F_z$ ,  $F_x$  and  $F_y$  acting on the cutter through the proper components acting on a tooth. Therefore, the purpose of this research work is to develop a mathematical model for the evaluation of face milling forces by combining the existing analytical models with experimental ones, and to consider the influences of the specific elements in face milling.

**Key words:** cutting force's components acting on a tooth, number of teeth that simultaneously cut, face milling forces.

### 1. Introduction

Theoretical and experimental studies related to the dynamics of face milling process aim the evaluation of cutting forces and torque, the influence

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that they have on the stability of technological system and also dynamometric constructions for their measurement. Forces in face milling have been extensively used also in studying other aspects of cutting process such as the manner that they can be used to control and monitor the cutting process, detecting wear and/or breaking of the cutting tools, construction and development of new types of cutters and milling machines (Deacu *et al.*, 1992), (Heikkala, 1995), (Minciu *et al.*, 1995).

Theoretical and experimental researches from the last years suggest the development of analytical models for cutting forces evaluation in face milling describing as realistic as possible the situations encountered in practice (Bissey *et al.*, 2005), (Korkut, 2007).

The improvement of valuation models for face milling forces is based on achieving a specific model for this variant of milling, which takes into account the factors influencing at one tooth's level and also the interdependencies between them, influences of the cutter's specific elements, the possible face milling variants (full and incomplete symmetrical milling, asymmetric routing), number of teeth that simultaneously cut and relative position between cutter and work-piece (cut - up and cut - down milling).

## 2. Theoretical Research in Developing New Valuation Models for Face Milling Forces

### 2.1. Elements of Mathematical Modeling for Cutting Forces in Face Milling

Fig. 1 presents the main factors which are considered in developing new valuation models for face milling forces.

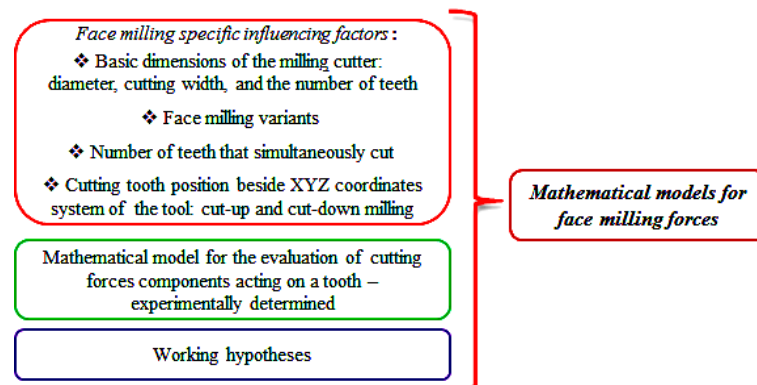


Fig. 1 – Elements of new analytical models for face milling forces.

Regarding the first set of influencing factors one can say that the basic dimensions of milling cutter are most often easily determined; the 5 possible variants of face milling were presented (Cozmîncă *et al.*, 2009), (Matei *et al.*, 2010) as they are described in the literature and with the improvements that we

considered necessary to achieve our goal in developing new valuation models for face milling forces for each variant – the possibility to process a material using 5 variants of face milling occurs as a result of the needs that arise from the limitations imposed by work-piece (size, material properties) or by cutter (diameter, number of cutting teeth), or due to the technical and economical requirements of the final product (the beneficiary); for each face milling variant, the number of teeth which simultaneously cut is calculated considering the number of teeth, cutter's diameter and depth of cut (Cozmîncă, 1995), (Cozmîncă *et al.*, 2009), (Matei *et al.*, 2010).

To evaluate the cutting force's components acting on a tooth we'll consider the analytical models in turning (Cozmîncă *et al.*, 2009), (Croitoru, 2000). These models were experimentally verified (Eqs. 1 – 4).

$$\begin{aligned}k_1 &= \cos \gamma_{N_0} (1 + \mu \cdot \operatorname{tg} \gamma_{N_0}) \\k_2 &= \cos \gamma_{N_0} (\mu - \operatorname{tg} \gamma_{N_0}) \\k_3 &= \sin \lambda \cdot \cos K \\k_4 &= \sin \lambda \cdot \sin K\end{aligned}\quad (1)$$

$$\begin{aligned}F_z &= F_N \cdot k_1 \cdot \cos \lambda \\F_x &= F_N (k_2 \cdot \sin K + k_1 \cdot k_3) \\F_y &= F_N (k_2 \cdot \cos K - k_1 \cdot k_4)\end{aligned}$$

$$\begin{aligned}C_1 &= k_1 \cdot \cos \lambda \\C_2 &= k_2 \cdot \sin K + k_1 \cdot k_3 \\C_3 &= k_2 \cdot \cos K - k_1 \cdot k_4\end{aligned}\quad (2)$$

$$\begin{aligned}F_z &= C_1 \cdot F_N \\F_x &= C_2 \cdot F_N \\F_y &= C_3 \cdot F_N\end{aligned}\quad (3)$$

$$F_N = \sigma_0 \cdot t \cdot s \cdot C_d^n \quad (4)$$

The ratios  $C_1$ ,  $C_2$  and  $C_3$  show the direct influence of angles  $\gamma$ ,  $\lambda$ ,  $K$ , chip's deviation angle ( $\eta = \lambda$ ), nature of material being cut (through friction coefficient  $\mu$  and the chips' contraction coefficient  $C_d$ ) and cross-sectional area of the chip ( $t \cdot s$ ) on force's components  $F_z$ ,  $F_x$  and  $F_y$  (Cozmîncă, 1995), (Cozmîncă *et al.*, 2009).

In order to use these relationships to determine the values of force's components in face milling we'll consider the simplifying assumptions (working hypotheses) described below:

- The force's components acting on a single tooth,  $F_z$ ,  $F_x$  and  $F_y$  are equal for all those  $z_s$  inserts that simultaneously cut, therefore, the

cutting force components take values according to the cross – sectional area of the chip, so we have  $F_z = F_{zmed}$ ,  $F_x = F_{xmed}$  and  $F_y = F_{ymed}$  for an average thickness of the chip  $a = a_{med}$ .

- Positioning and orientation of the cutter's coordinate system  $XYZ$  and  $xyz$  system of the cutting tooth.

Regarding the second working hypothesis one can say that the new valuating models for the components  $F_z$ ,  $F_x$  and  $F_y$  of the resulting cutting force  $F_r$  acting on the cutter, depend on the positioning and orientation elements from Fig. 2.

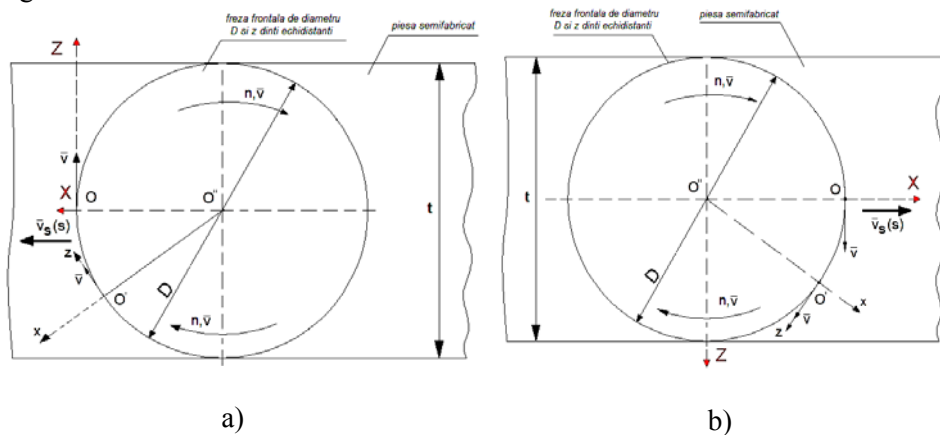


Fig. 2 – Positioning and orientation of the coordinates systems  $XYZ$  and  $xyz$  in face milling; Conventional face milling (a); Climb face milling (b).

The origin  $O$  of the cutter's coordinates system  $XYZ$  is chosen in the cutter's point where the feed direction ( $\vec{v}_s$ ) corresponds to the cutter's radial direction. The origin  $O'$  of the tooth's coordinate's system  $xyz$  is positioned in the peak of the tooth which describes the circle with radius equal to  $D/2$ .

The orientation of  $Z$  and  $z$  axes corresponds to main cutting speed vector  $\vec{v}$ ,  $X$  axis corresponds to feed direction ( $\vec{v}_s$ ),  $x$  axis corresponds to radial direction  $O'O''$  and  $Y$  and  $y$  axes are parallels to the cutter's rotation axis.

Since the resulting cutting force  $F_r$  acting on the cutter and cutting force  $F_d$  acting on a tooth are oriented from cutter to work-piece, between the positive orientations of the axes  $ZX$  and  $zx$  and the components  $F_z$ ,  $F_x$ ,  $F_y$  and  $F_z$ ,  $F_x$ ,  $F_y$  orientations there are no differences according to Fig. 3.

The components  $F_y$  and  $F_Y$  are perpendicular to the figure and parallels to cutter's axis. We proposed the assimilation of tangential, radial and axial components acting on a tooth with  $F_z$ ,  $F_x$  and  $F_y$  components from turning. This hypothesis will determine that the values of  $F_z$ ,  $F_x$  and  $F_y$  components acting on cutter to be in the well – known ratio of turning, respectively,  $F_z > F_x > F_y$ .

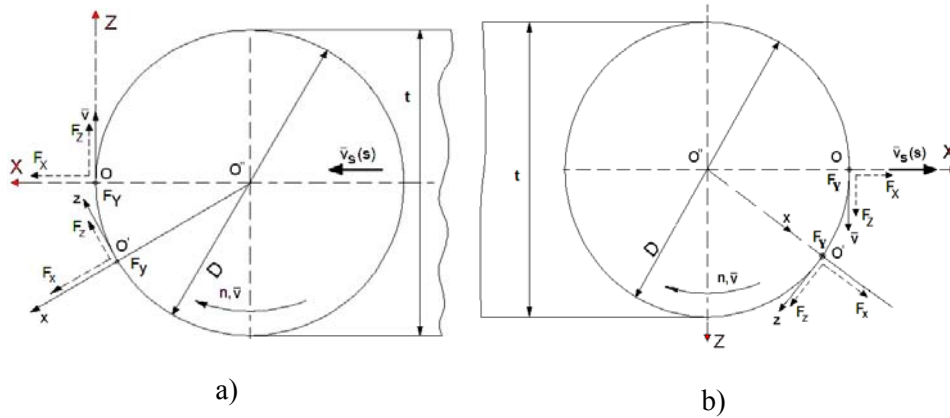


Fig. 3 – The orientation of cutting force’s components in the normal plane to cutter’s axis; (a) Conventional face milling; (b) Climb face milling.

### 2.2. New Valuation Models for Cutting Force’s Components in Face Milling

The new mathematical models proposed to evaluate the cutting force’s components in face milling were reduced to 3 sets of equations, namely Eqs. (4) - (6) (Matei, 2012).

For *asymmetrical face milling with  $\Psi \leq 90$*  (Figs. 4 and 5) we can have either cut – up or cut – down milling and eqs, (4) and (5) were developed depending on the relative position between cutter and work-piece.

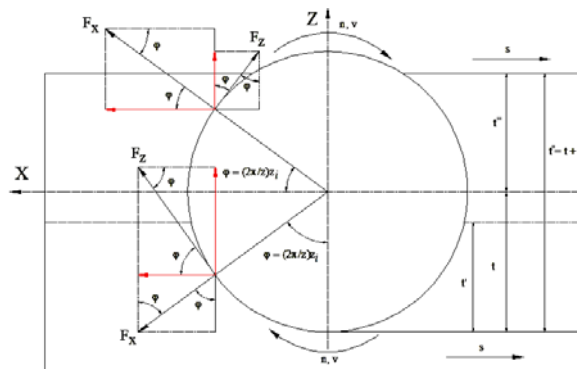


Fig. 4 – Cutting force’s components in conventional asymmetrical face milling.

a) **cut – up milling**

$$F_Z = \sum_1^{z_s} F_{z_{med}} \sin\left(\frac{2\pi}{z} z_{s_i}\right) \quad (4)$$

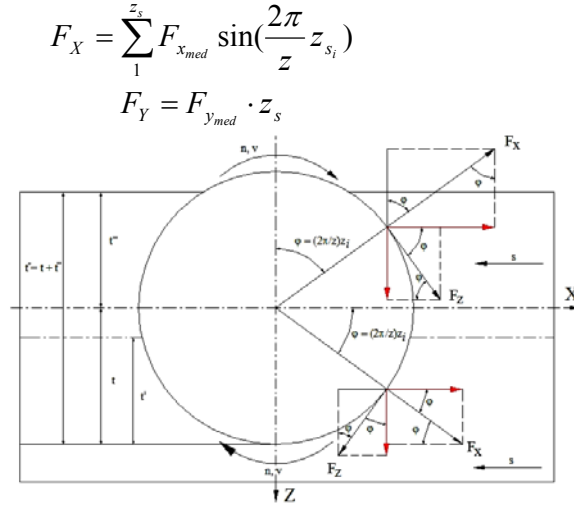


Fig. 5 – Cutting force's components in climb asymmetrical face milling.

b) **cut – down milling**

$$F_Z = \sum_1^{z_s} F_{z_{med}} \cos\left(\frac{2\pi}{z} z_{s_i}\right)$$

$$F_X = \sum_1^{z_s} F_{x_{med}} \cos\left(\frac{2\pi}{z} z_{s_i}\right) \quad (5)$$

$$F_Y = F_{y_{med}} \cdot z_s$$

For *asymmetrical face milling with  $\Psi > 90^\circ$  and symmetrical face milling* (Figs. 4–7) the cut – up and cut – down milling are combined, predominating one of them (for the asymmetrical milling with  $\Psi > 90^\circ$ ), and equations (6) were developed.

$$F_Z = \sum_1^{z_s} F_{z_{med}} \sin\left(\frac{2\pi}{z} z_{s_i}\right) + \sum_1^{z_s} F_{z_{med}} \cos\left(\frac{2\pi}{z} z_{s_i}\right)$$

$$F_X = \sum_1^{z_s} F_{x_{med}} \sin\left(\frac{2\pi}{z} z_{s_i}\right) + \sum_1^{z_s} F_{x_{med}} \cos\left(\frac{2\pi}{z} z_{s_i}\right) \quad (6)$$

$$F_Y = F_{y_{med}} \cdot z_s$$

In order to develop the valuation models for force's components in these two cases we considered that the cutter is processing both in climb and conventional milling, the positioning of the cutter beside work-piece being very important. One can observe that in symmetrical face milling equations similar to the one for asymmetrical face milling with  $\Psi > 90^\circ$  were found, differing only by the number of teeth which simultaneously cut. In all the cases the number of

teeth  $z_s$  that simultaneously cut varies from one milling variant to another and it can be calculated using the models proposed (Cozmîncă *et al.*, 2009), (Matei *et al.*, 2010).

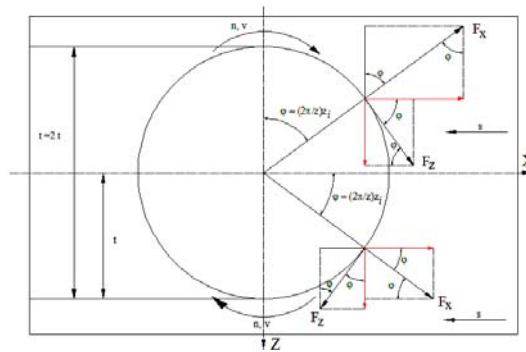


Fig. 6 – Cutting force's components in climb symmetrical full face milling.

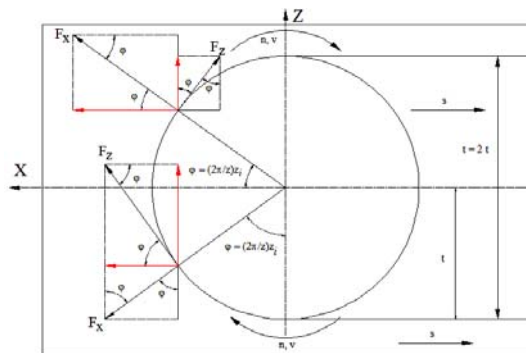


Fig. 7 – Cutting force's components in conventional symmetrical full face milling.

### 3. Conclusions

Using these analytical models in technological designing requires knowing the working conditions (cutting conditions, cutting tooth geometry and the nature of material being cut). Knowing all these we can first calculate the values of components  $F_{zmed}$ ,  $F_{xmed}$  and  $F_{ymed}$  acting on a tooth using the equations (3). In support of this, the average chip thickness is evaluated using the adopted working feed, then we calculate the values for  $C_1$ ,  $C_2$  and  $C_3$  depending on the constructive angles of tooth (Eqs. 1–3.). Next we evaluate the deformation force  $F_N$  with the well-known equation  $F_N = \sigma_0 \cdot t \cdot s \cdot C_d^n$ , where  $\sigma_0$  is the standard compression yield point of the material, for  $C_d$  we can use both theoretical and experimental values, and  $n = 1.5$  according to the literature in this area (Cozmîncă, 1995), (Minciu, 1995). In further papers we propose to

experimentally verify the way that these relationships can be applied in real situations

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### CONTRIBUȚII TEORETICE LA DEZVOLTAREA UNOR NOI MODELE DE EVALUARE A FORȚELOR LA FREZAREA FRONTALĂ

(Rezumat)

Prin această lucrare ne propunem îmbunătățirea modelelor de evaluare a forțelor la frezarea frontală astfel încât toate influențele semnificative să fie prezente la adevărata lor valoare. În acest sens se vor avea în vedere, în principal, componentele forței de aşchiere de la nivelul unui dinte și numărul de dinți care aşchiază simultan. Condițiile de lucru, respectiv regimul de aşchiere, geometria dintelui, natura materialului aşchiat și mediul de aşchiere, sunt prezente în  $F_z$ ,  $F_x$  și  $F_y$  de la nivelul frezei prin intermediul componentelor corespunzătoare de la nivelul unui dinte aşchietor. Ca urmare, scopul lucrării este acela de a realiza un model matematic de evaluare a forțelor la frezarea frontală care să îmbine modelele analitice actuale cu cele experimentale și să ia în considerare influențele elementelor specifice frezării frontale: varianta de frezare posibilă, numărul de dinți care aşchiază simultan, mărirea forțelor de la nivelul unui dinte și poziția relativă a dinților față de sistemul de coordonate al frezei.